Chapter 4

IRREDUCIBLE CONTINUA

4.1 Elementary examples

(a) An arc, Sin Curve

(b) Other examples (Cantor Organ, Cantor Accordion, etc.)

4.1.1 Sin curve

The *(topologist's) sin curve* S is defined by

$$S = \{ (x, y) \in \mathbb{R}^2 : y = \sin \frac{1}{x}, 0 < x \leq 1 \} \cup \{ (0, y) : -1 \leq y \leq 1 \}.$$

1. S is irreducible between points $(0, y)$ and $(1, \sin 1)$, $-1 \leq y \leq 1$. It has exactly three end points and two arc components.

2. It is one of the simplest arc-like continua.

3. It is a compactification of a ray $(0, 1]$ with remainder an arc.

4. It has the periodic-recurrent property [Charatonik et al. 1997b, Corollary 5.10, p. 117].

5. The only possible confluent nondegenerate images of S are an arc and a continuum homeomorphic to S [Nadler 1977a].

6. The hyperspace $C(S)$ of all subcontinua of S is homeomorphic to the cone over S [Nadler 1977b].

See Figure A.

There are many variations of the sin curve. Some of them are pictured below. See Figure B–C.