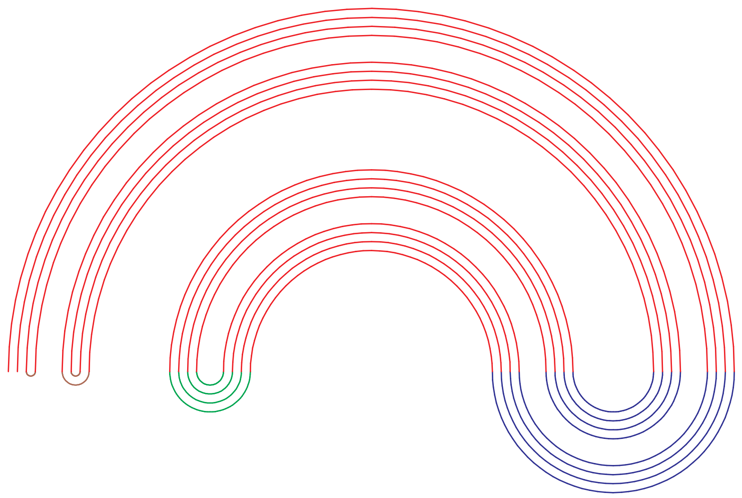

Figure 3.2.1: ( A ) buckethandle

Nice geometric models of $B_1$ and $B_2$ can be described in the following way (see [Kuratowski 1968, pp. 204–205]). If $C$ denotes the standard Cantor ternary set in $I$, then $B_1$ is homeomorphic to the union of all semi-circles

$$\{ (x,y) \in \mathbb{R}^2 : (x - \frac{1}{2})^2 + y^2 = r^2, y \geq 0 \},$$

where $r + \frac{1}{2} \in C$, and all semi-circles

$$\{ (x,y) : (x - \frac{5}{2 \cdot 3^n})^2 + y^2 = r^2, y \leq 0 \},$$

for $n \in \mathbb{N}$, where $r + \frac{5}{2\cdot 3^n} \in C \cap [\frac{2}{3^n}, \frac{1}{3^{n-1}}]$.

See Figure A.

If $E$ is the quintary Cantor set in $I$ (real numbers in $I$ which can be written in the enumeration system at base 5 without digits 1 and 3), then $B_2$ is homeomorphic to the union of all semi-circles

$$\{ (x,y) : (x - \frac{7}{10 \cdot 5^n})^2 + y^2 = r^2, y \leq 0 \},$$

for $n \in \mathbb{N}$ and $r + \frac{7}{10\cdot 5^n} \in E \cap [\frac{2}{5^{n+1}}, \frac{1}{5^n}]$ and all semi-circles

$$\{ (x,y) : (x - (1 - \frac{7}{10 \cdot 5^n}))^2 + y^2 = r^2, y \geq 0 \},$$

for $n \geq 0$ and $\frac{2}{5^{n+1}} \leq \frac{7}{10\cdot 5^n} - r \leq \frac{1}{5^n}$.

See Figure B.