1.3.14 Miller dendrite

Recall that a space is said to be incompressible if it is homeomorphic to no proper subspace of itself. The circle S^1 is an example of such a space. In 1925 Zarankiewicz [Zarankiewicz 1925] asked whether there exists an incompressible dendrite. The question has been answered in the positive in 1932 by E. W. Miller [Miller 1932]. The reader is referred to the original Miller's paper for the (rather long) construction of the example. The main idea of the proof is based on the following implication, [Miller 1932, Theorem, p. 831].

If a dendrite X contains a subset K such that

A. each point of K is fixed with respect to any homeomorphism of X onto a subcontinuum of X;

B. each point of X which is not an end point of X lies in an arc contained in X and having its end points in K,

then X is incompressible.

The *Miller dendrite* S has the following properties, [Miller 1932].

1. S contains a subset K having properties A and B above, so it is incompressible.

2. S is not chaotic.

3. S contains open arcs as open subsets, hence it is not rigid.

1.3.15 Dendrites of de Groot-Wille type

An example of a rigid dendrite has been roughly described in [J. de Groot et al. 1958]. Its construction is recalled below (see also [Charatonik 1979, Chapter III, Section 5 , p. 227]).

For any $i \in \mathbb{N}$ let T_i denote the i-od being the union of i straight line unit segments emanating from a point called the origin of T_i. We proceed by induction. Define Y_0 as the unit segment. Let x be the midpoint of Y_0, and define Y_1 as the union of Y_0 and a diminished copy of $T_{j_1} = T_1$ so that the diameter of this copy is less than $1/2$ and that $Y_0 \cap T_1 = \{x\}$. Assume that a tree Y_n is defined such that it is the union of finitely many straight line segments and contains (properly diminished) copies of $T_1, T_2, \ldots, T_{j_n}$, i.e., of the first j_n terms of the sequence $\{T_i\}$. To define Y_{n+1} consider all maximal free segments in Y_n. Note that there are finitely many, say $m(n)$ of them. Let x denote the midpoint of any of these segments. With each point x so defined we associate, in a one-to-one way, a set T_i taken from the $m(n)$ consecutive terms of the sequence $\{T_i\}$, i.e., we use in this step of the construction the sets $T_{j_n+1}, T_{j_n+2}, \ldots, T_{j_{n+1}}$, where $j_{n+1} = j_n + m(n)$. We take each midpoint as the origin of a properly diminished copy of T_i, where $i \in \{j_n + 1, j_n + 2, \ldots, j_{n+1}\}$ in such a way that the diameter of the copy of T_i is less than $1/2^{n+1}$ and that Y_n has only the