1.3.9 The dendrite P

In connection with the characterization of dendrites having closed set of their end points (see Property 2 in 1.3.4 above) the classes of those dendrites that do not contain a copy of F_ω and those that do not contain a copy of the comb W are of some interest (see [Arévalo et al. 2001, p. 8]). The first one is just the class of dendrites with finite orders of ramification points, and it is known to have a universal element according to Property 4 in 1.3.8. The other class has also a universal element P. Its construction, given in [Arévalo et al. 2001, p. 9] is the following.

Let p be any point in the plane $\mathbb{R}^2$ and let $p_1, p_2, \ldots$ be a sequence of points in $\mathbb{R}^2$ tending to p and such that no three of the points $p, p_1, p_2, \ldots$ are collinear. Define $P_1 = \bigcup\{\overline{pp_n} : n \in \mathbb{N}\}$. Then P_1 is homeomorphic to F_ω. For every $n, m \in \mathbb{N}$, let p_{nm} be a point in $\mathbb{R}^2 \setminus P_1$ such that for any fixed n the sequence $p_{n1}, p_{n2}, \ldots$ tends to p_n, no three of the points $p_n, p_{n1}, p_{n2}, \ldots$ are collinear, and that the continuum $P_2 = P_1 \cup \bigcup\{\overline{p_n p_{nm}} : n, m \in \mathbb{N}\}$ is a dendrite. We continue constructing dendrites $P_3, P_4, \ldots$ in the same manner, such that we get an increasing sequence of dendrites $P_1 \subset P_2 \subset P_3 \subset \cdots \subset P_n \subset P_{n+1} \subset \ldots$ in $\mathbb{R}^2$ having the property that the closure of their union is also a dendrite. Finally define

$$P = \operatorname{cl}\left(\bigcup\{P_n : n \in \mathbb{N}\}\right).$$

The following properties of the *dendrite* P are proved in [Arévalo et al. 2001, Theorems 4.5 and 4.6, p. 10].

1. A dendrite X is homeomorphic to the dendrite P if and only if each of the following conditions is satisfied:
   (a) X contains no copy of W;
   (b) the set of all end points of X is contained in the closure of the set of all ramification points of X;
   (c) all ramification points of X are of order ω.
2. The dendrite P is universal in the class of all dendrites containing no copy of W.

See Figure A.

1.3.10 Self-homeomorphic dendrites

Below we present some examples of dendrites that illustrate relations between variants of self-homeomorphic properties.

1. There is a strongly pointwise self-homeomorphic dendrite which is not of the form D_S for some $S \subset \{3, 4, \ldots, \omega\}$, see [Charatonik W.J. et al. 1994, Example 6.11, p. 232, and Fig. 4, p. 233]. Note that, according to Property 1 in 1.3.8 each dendrite D_S is