Examples in Continuum Theory

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Contents

Chapter 1

Arc-like continua

1.1 Topologists's Sin Curve

The *Topologist's Sin Curve* is well known continuum which is not arcwise connected. The continuum is defined by the union

$$\{(x, y) \in R^2 : 0 < x \leq 1, y = \sin(1/x)\} \cup \{(0, y) \in R^2 : -1 \leq y \leq 1\} \quad .$$

It has just two path components. More we can find in [74, p.137] .

1.2 Open Sin

The *Open Sin* is defined by the Euclidean closure of the set

$$\{(x, y) \in R^2 : -1 < x < 1, y = \sin(1/(x^2 - 1))\} \quad .$$

This continuum has just three path components.

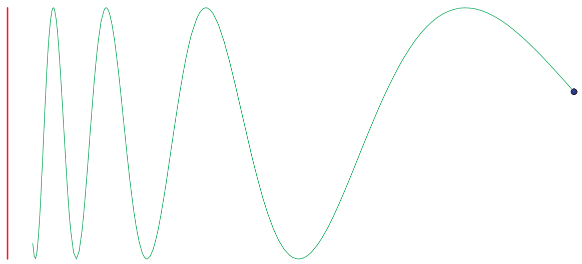

Figure 1.1: Topologists's Sin Curve- fig. a

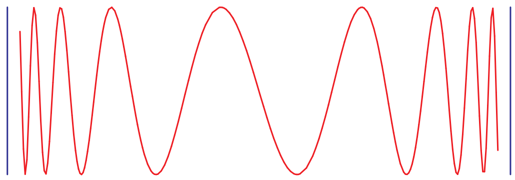

Figure 1.2: Open Sin- fig. a

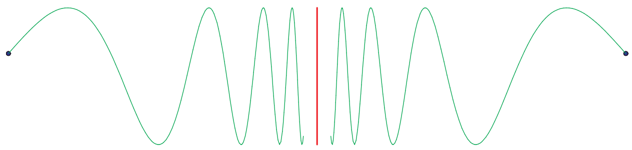

Figure 1.3: Double Topologists's Sin Curve- fig. a

1.3 Double Topologists's Sin Curve

The *Double Topologists's Sin Curve* is defined by the union

$$\{(x, y) \in R^2 : 0 < |x| \leq 1, y = \sin(1/x)\} \cup \{(0, y) \in R^2 : -1 \leq y \leq 1\} \quad .$$

It has just three path components.

1.4 Snake

The *Snake* is an arclike continuum where each open mapping is a homeomorphism.

We can find more in [40, Theorem ..].

1.5 Cantor Snake

The *Cantor Snake* X is formed by combining the product $C \times I$ of the Cantor Ternary Set C and the unit interval I with countably many Open Sin Curves in such a way that the space X has uncountably many arc components.

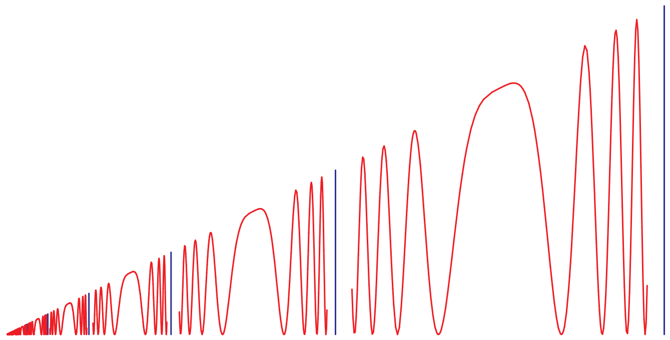

Figure 1.4: Snake- fig. a

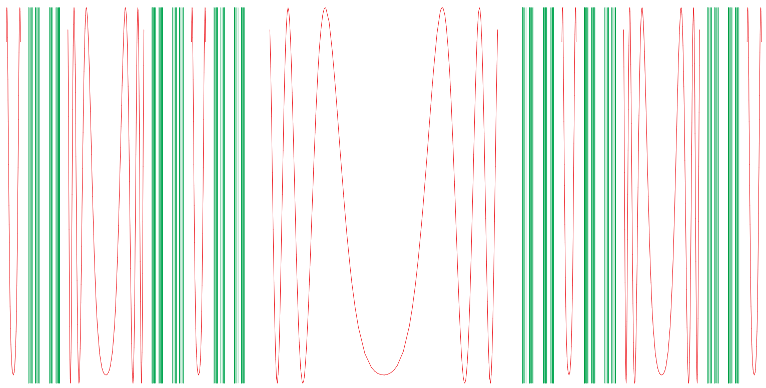

Figure 1.5: Cantor Snake- fig. a

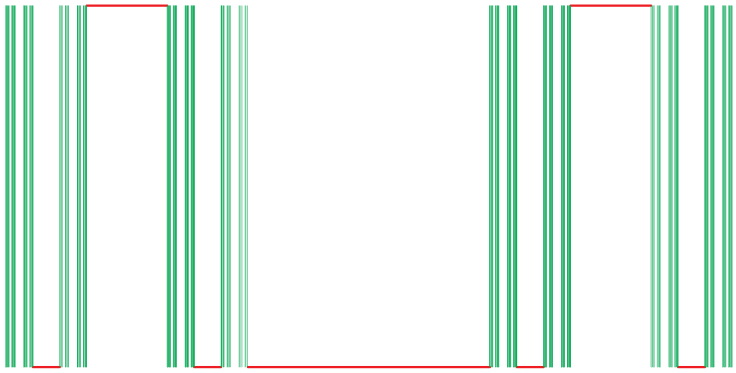

Figure 1.6: Cantor Meanders- fig. a

1.6 Cantor Meanders

The *Cantor Meanders* X is formed by combining the product $C \times I$ of the Cantor Ternary Set C and the unit interval I with the appropriate segments of $I \times \{0, 1\}$. We use the 'zig-zag' system of selecting the intervals. We start with $[1/3, 2/3] \times \{0\}$, continue with $[1/9, 2/9] \times \{1\}$ and $[7/9, 8/9] \times \{1\}, \cdots$.

The space X has uncountably many arc components.

1.7 Buckethandle

The *Buckethandle* is created from the Cantor Ternary Set C with this procedure:

(i) we join any two points a and b in C symmetric with respect to $1/2$ with a semicircle in the upper half plane with the centre in $(1/2, 0)$,

(ii) we join any two points of C in the interval $2/3^n \leq x \leq 3/3^n$, $n \geq 1$, with a semicircle with the centre in $(5/(2 \cdot 3^n), 0)$ in the lower half plane.

This continuum is often called the *Knaster's Buckethandle* continuum.

We can obtain a homeomorphic copy of the buckethandle continuum using the inverse limit

$$\varprojlim \{X_i, f_i\}_{i=1}^{\infty}$$

where for each $i = 1, 2, \cdots$ let $X_i = [0, 1]$ and $f_i(t) = 2t$ for $0 \leq t \leq 1/2$ and $f_i(t) = -2t + 2$ for $1/2 \leq t \leq 1$.

We can find more in [65, p.22] and [55, p.205].

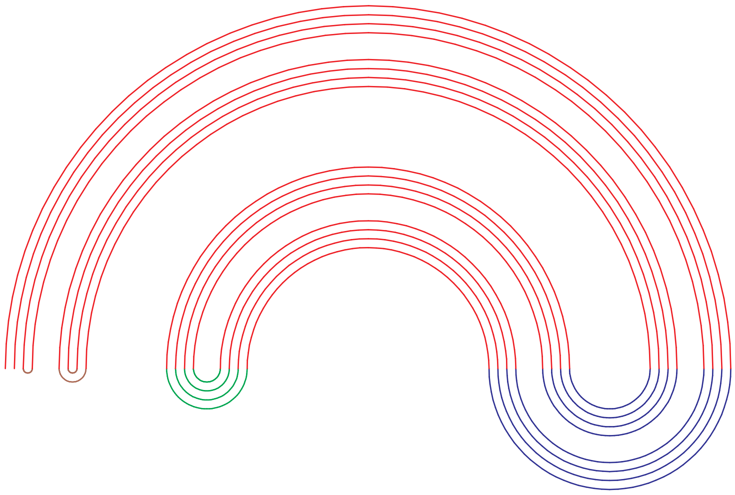

Figure 1.7: Buckethandle - fig. a

1.8 Double Buckethandle

The *Cantor* 1/5-*discontinuum* C_5 is created using the procedure of deleting the second and the fourth segments from five equal segments (instead of the middle third for the Cantor Ternary Set). C_5 is the set of the reals in the unit interval which can be represented without 1-s and 3-s in the form

$$\sum_{n=1}^{\infty} \frac{a_n}{5^n} \quad .$$

The *Double Buckethandle* continuum is the union of:

(i) the half circles in the lower plane going through points of $C_5 \cap \{x : 2/5^{n+1} \leq x \leq 1/5^n\}$ with the centre $7/10 \cdot 5^n$,

(ii) the half circles in the upper plane going through points of $C_5 \cap \{x : 2/5^{n+1} \leq 1 - x \leq 1/5^n\}$ with the centre $1 - 7/10 \cdot 5^n$.

We can find more in [55, p.205 - p.206].

1.9 Cruller

The *Cruller* continuum is beautiful.

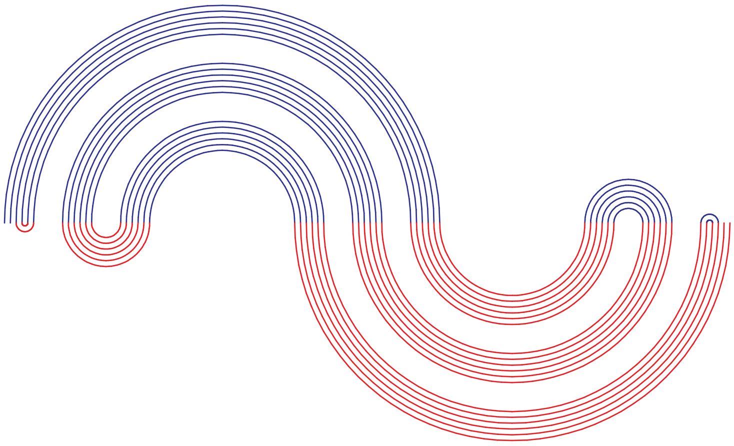

Figure 1.8: Double Buckethandle - fig. a

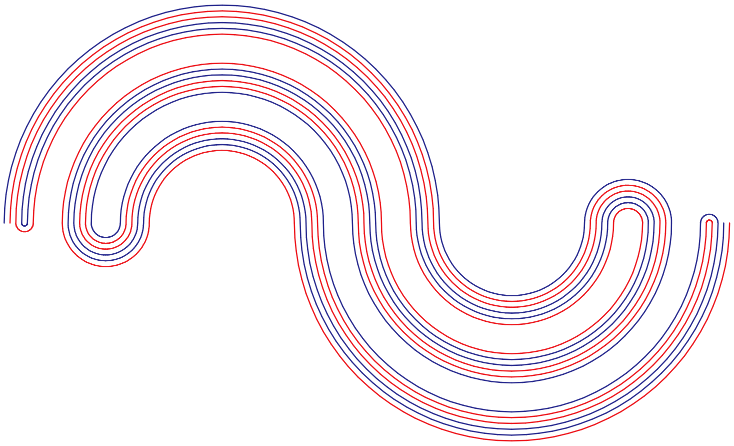

Figure 1.8: Double Buckethandle - fig. b

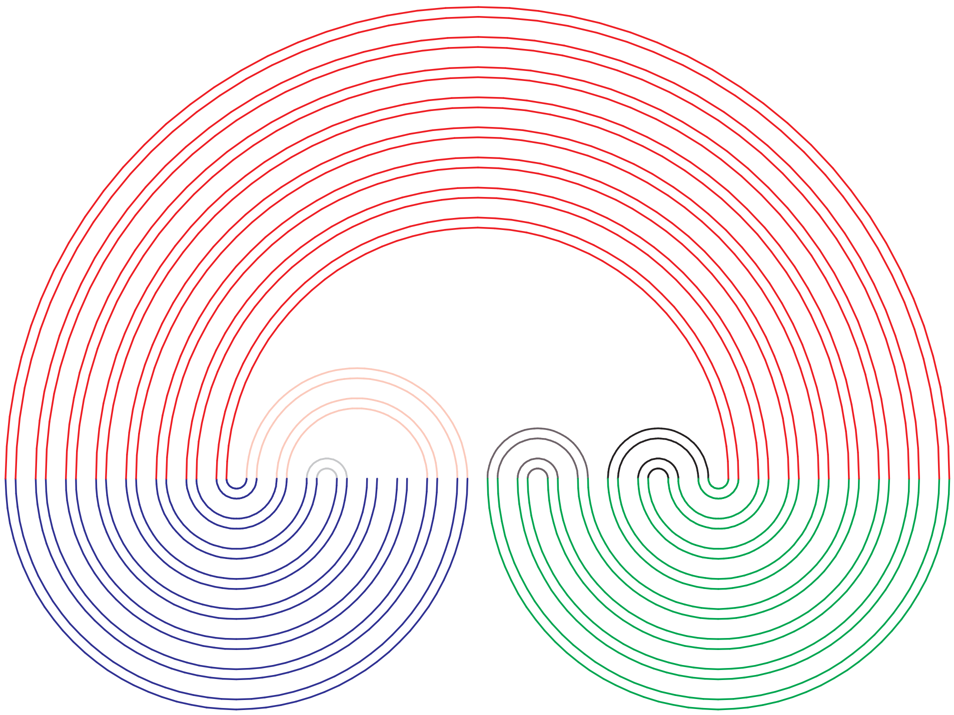

Figure 1.9: Cruller- fig. a

Chapter 2

Self-homeomorphic continua

2.1 Sierpiński Universal Plane Curve

The *Sierpiński Universal Plane Curve* is a well known continuum which serves as the universal element in the class of all one-dimensional continua in the plane. It is obtained as the residual set remaining when one begins with a square and applies the operation of dividing it into nine equal squares and omitting the interior of the center one, then repeats this operation on each of the surviving 8 squares, then repeats again on the surviving 64 squares, and so on $\cdots$. See [65, p.9].

The Sierpiński Universal Plane Curve can be characterized as the only plane locally connected one-dimensional continuum S such that the boundary of each complementary domain of S is a simple closed curve and no two of these complementary domain boundaries intersect. See [79].

2.2 Menger Universal Curve

The *Menger Universal Curve* X is obtained using the Sierpiński Universal Plane Curve S from the unit cube I^3 by the formula

$$X = \{(x, y, z) \in I^3 : (x, y) \in S \quad \& \quad (x, z) \in S \quad \& \quad (y, z) \in S\} \quad .$$

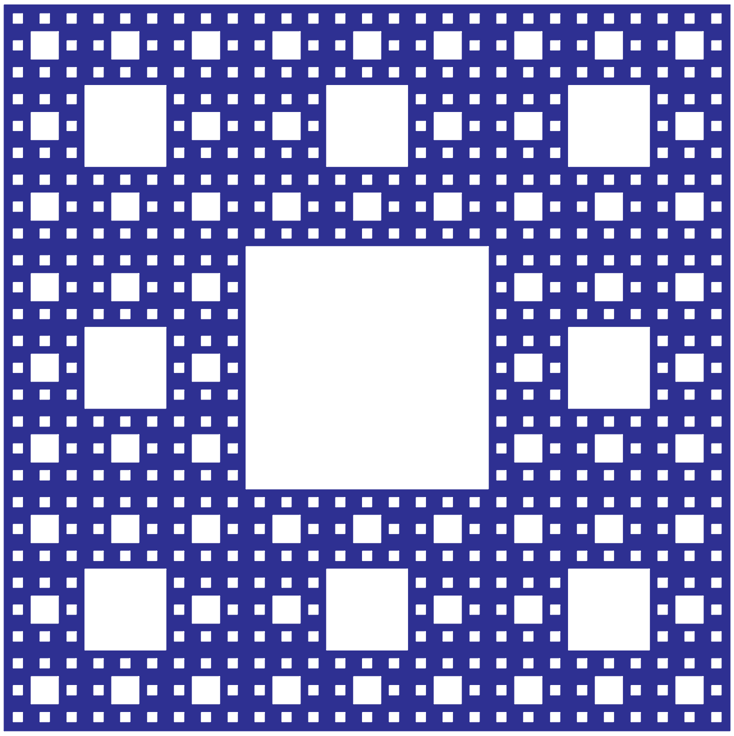

Figure 2.1: Sierpiński Universal Plane Curve- fig. a

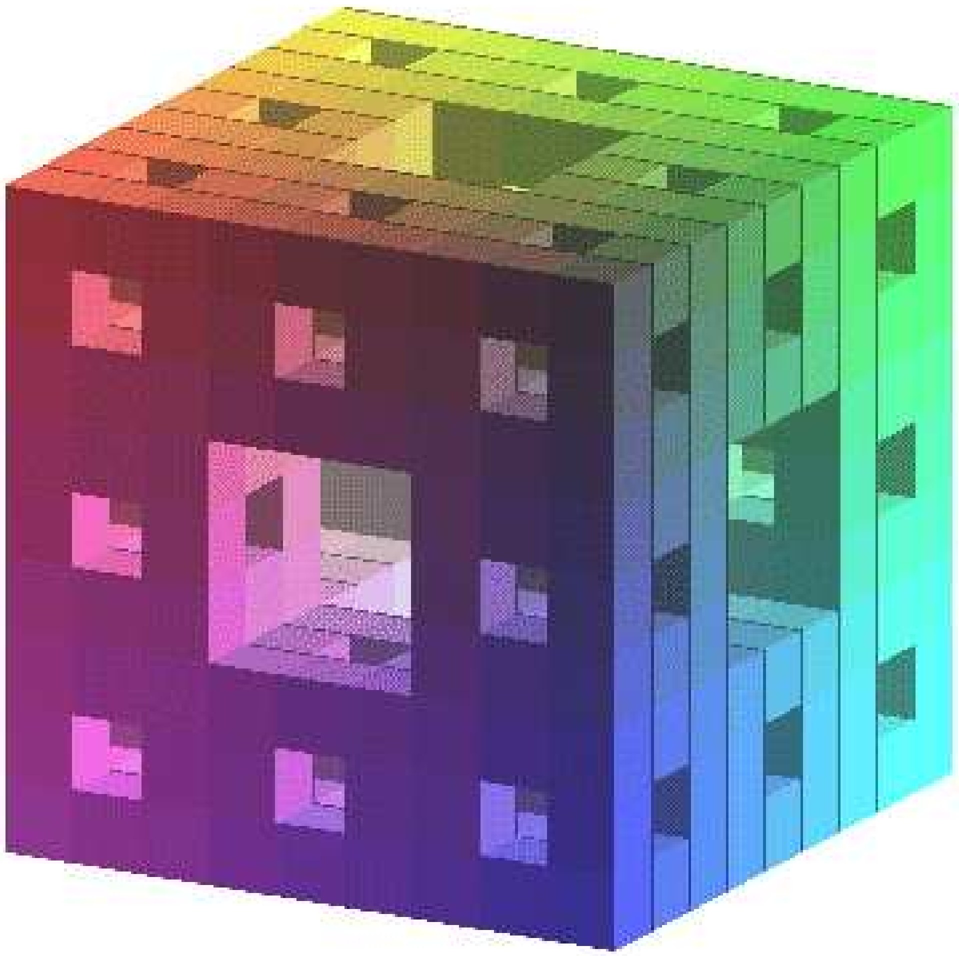

Figure 2.2: Menger Universal Curve- fig. a

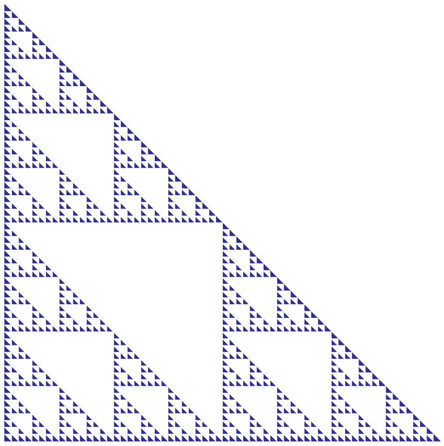

Figure 2.3: Sierpinski Triangle- fig. a

2.3 Sierpinski Triangle

The *Sierpinski Triangle* is obtained as the residual set remaining when one begins with a triangle and applies the operation of dividing it into four equal triangles and omitting the interior of the center one, then repeats this operation on each of the surviving 3 triangles, then repeats again on the surviving 9 triangles, and so on $\cdots$. See [36, Example 2.7].

The Sierpinski Triangle is homeomorphic to the unique nonempty compact set K of the complex plane that satisfies

$$K = w_1(K) \cup w_2(K) \cup w_3(K) \quad ,$$

where w_1, w_2, w_3 are maps of the complex plane defined by $w_1(z) = z/2$, $w_2 = (z+1)/2$ and $w_3 = (z+i)/2$.

2.4 Root

The *Root* is the continuum obtained attaching small 'T' letters to the previous ones as in the figure. It is a strongly pointwise self-homeomorphic dendrite. It has points of order three on the horizontal segments and points of order four on the vertical segments.

See [36, p.232-233].

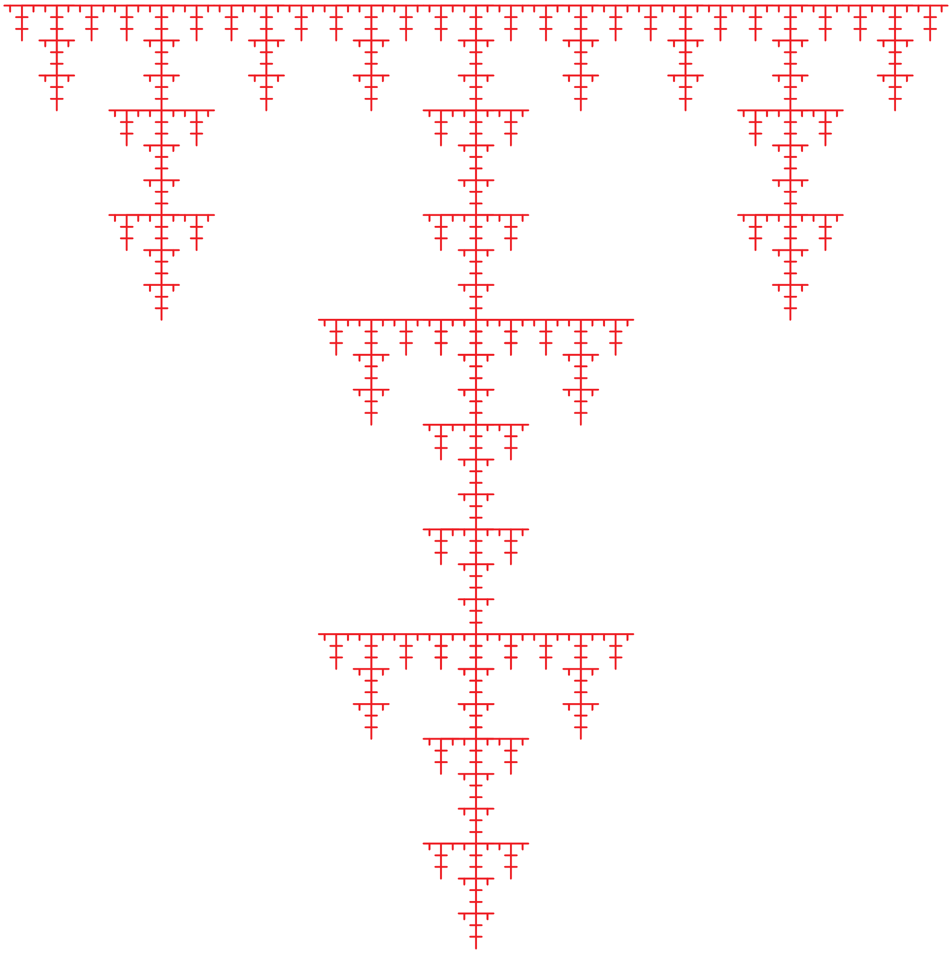

Figure 2.4: Root- fig. a

Chapter 3

Dendrites

3.1 Wazewski's Universal Dendrite

The *Wazewski's Universal Dendrite* is well known continuum which serves as the universal element in the class of all dendrites. We start with a Locally Connected Fan F_ω. Denote $D_1 = F_\omega$. Now we attach sufficiently small copies of F_ω to all midpoints of all maximal free arcs in D_1 and obtain a dendrite D_2. Continuing in this fashion, we obtain the sequence of dendrites $\{D_n\}$ and the Wazewski's Universal Dendrite $\mathbf{D}$ is the closure of their union. For details see [65, p.181-185].

3.2 Universal Dendrite $\mathbf{D_3}$

The *Universal Dendrite* $\mathbf{D_3}$ is obtained by the same procedure as the Wazewski Universal Dendrite just instead of locally connected fans we use simple triods.

We start with a simple triod T. Denote $T_1 = T$. Now we attach sufficiently small copies of T (with one end point as the attaching point) to all midpoints of all maximal free arcs in T_1 and obtain a dendrite T_2. Continuing in this fashion, we obtain the sequence of dendrites $\{T_n\}$ and the Universal Dendrite $\mathbb{D}_3$ is the closure of their union.

The Universal Dendrite $\mathbf{D_3}$ is homogeneous with respect to the monotone mappings. For details see [38, p.362].

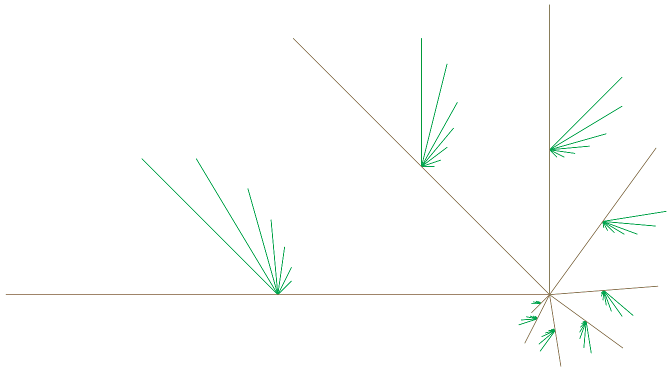

Figure 3.1: Wazewski's Universal Dendrite- fig. a

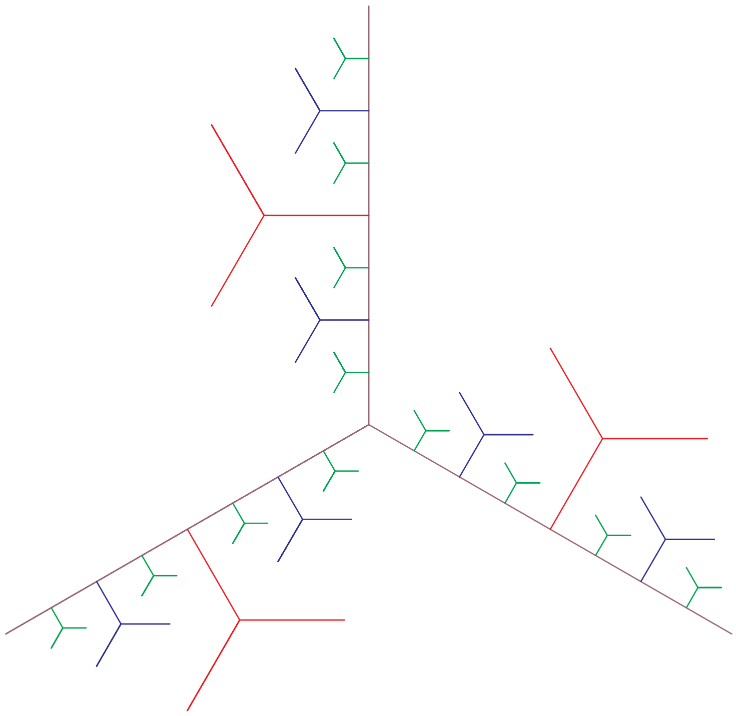

Figure 3.2: Universal Dendrite $\mathbf{D_3}$- fig. a

3.3 Universal Dendrite $\mathbf{D_4}$

The *Universal Dendrite* $\mathbf{D_4}$ is a dendtrite with ramification points of order 4.

3.4 Universal Dendrite $\mathbf{D_6}$

3.5 Gehman Dendrite

The *Gehman Dendrite* G is formed from the Cantor Ternary Set C in such a way that C is the set of end points of G and all ramification points of G are of order three.

See [67, pp. 422-423] and [68, p. 203].

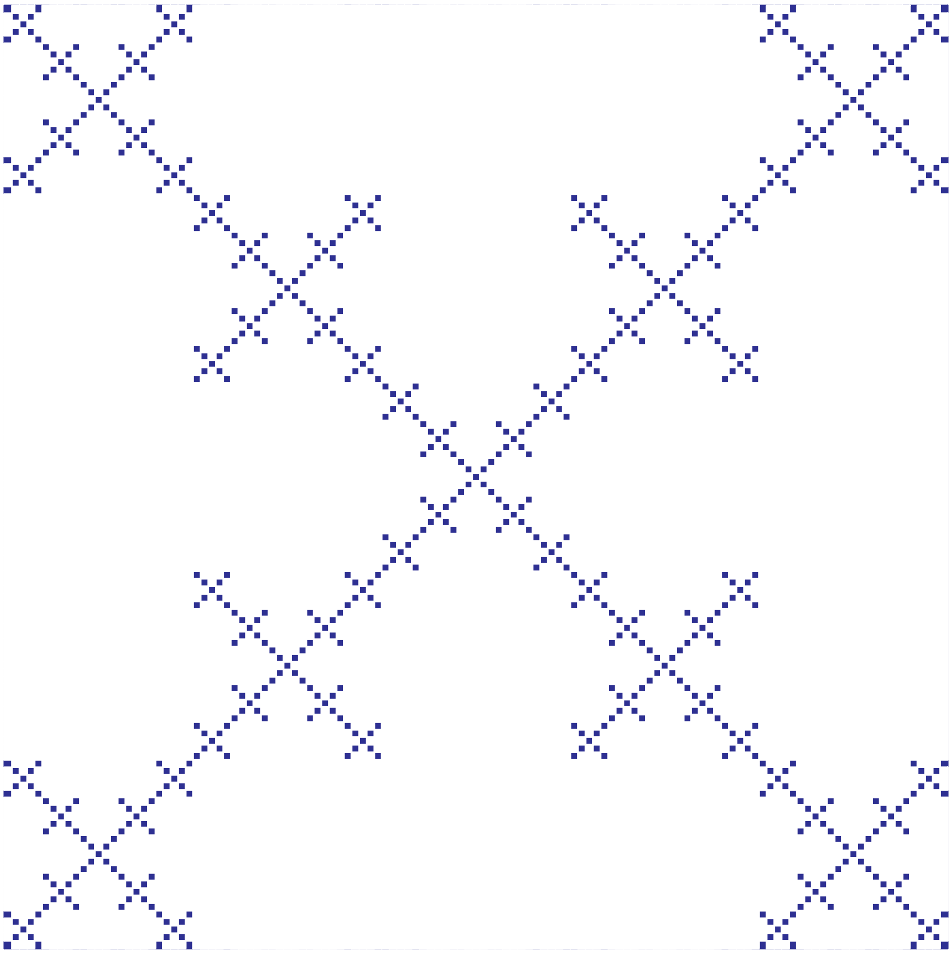

Figure 3.3: Universal Dendrite $\mathbf{D_4}$- fig. a

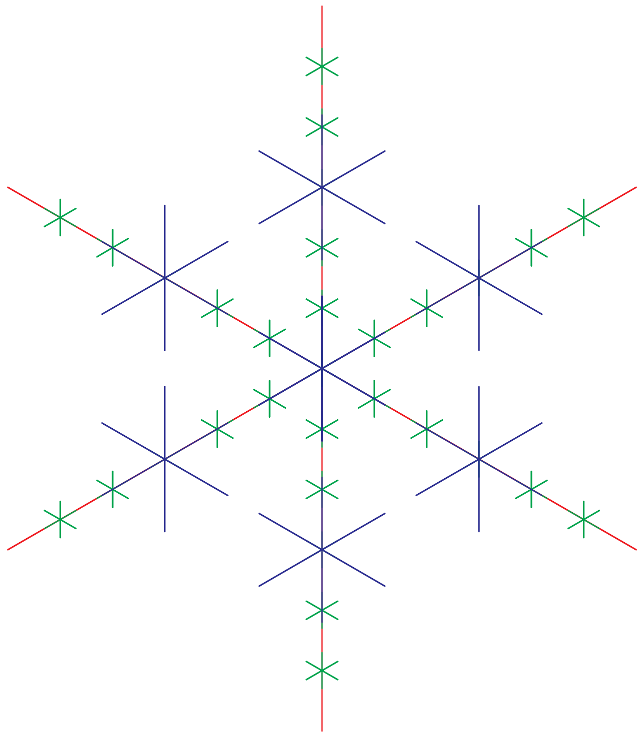

Figure 3.4: Universal Dendrite $\mathbf{D_6}$- fig. a

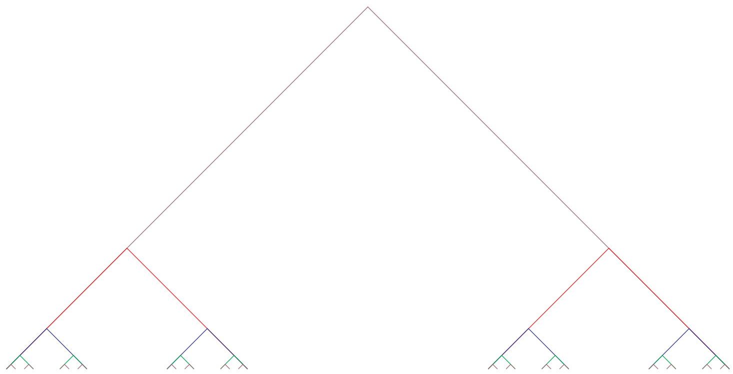

Figure 3.5: Gehman Dendrite- fig. a

Chapter 4

Dendroids

4.1 Cantor Comb

The *Cantor Comb* X is formed from the Cantor Ternary Set C and the unit interval I using the formula

$$X = C \times I \quad \cup \quad I \times \{0\} \quad .$$

4.2 Gehman Dendrite with Ski

The *Gehman Dendrite with Ski* G_s is a dendroid formed from the Gehman Dendrite G and the Cantor Ternary Set C with a formula

$$G_s = \{(x, y, z) \in R^3 : (x, y) \in G\} \cup \{(x, y, z) \in R^3 : x \in C, y = 0, |z| \leq 1\} \quad .$$

In fact we attach uncountably many segments of length 2 with their midpoints to the points of Cantor Ternary Set (the end points of Gehman Dendrite). See [25, p. 16].

4.3 Makuchowski Twin

We obtain the *Makuchowski Twin* as a subset of the Euclidean plane with the induced topology by attaching the Locally Connected Fan to the end point of the prolonged segment of the Prolonged Harmonic Fan.

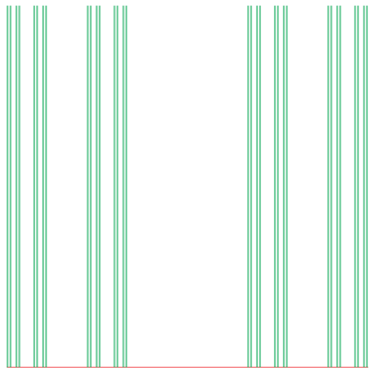

Figure 4.1: Cantor Comb- fig. a

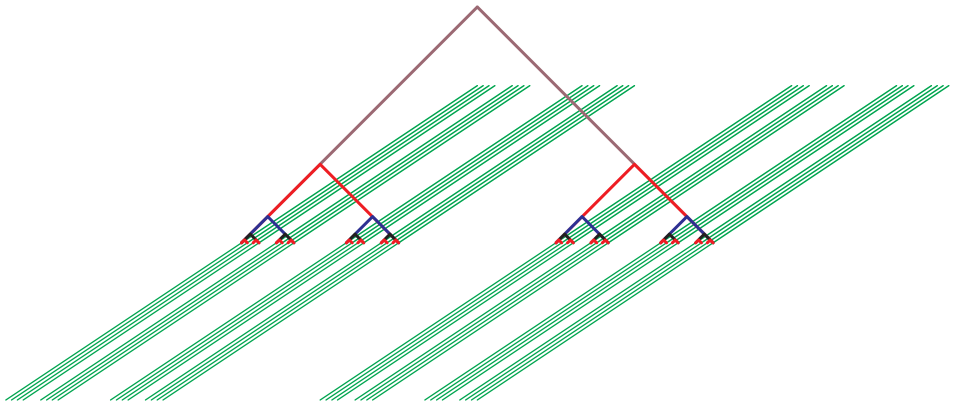

Figure 4.2: Gehman Dendrite with Ski- fig. a

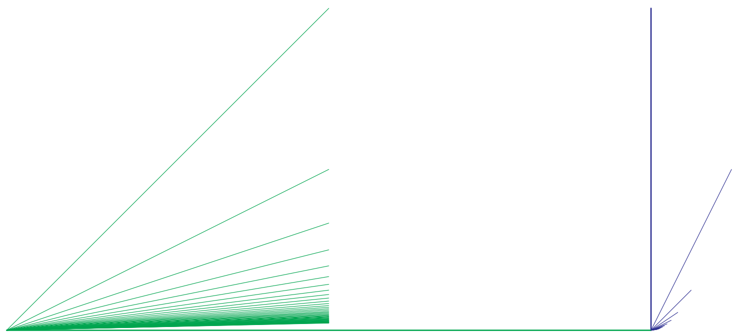

Figure 4.3: Makuchowski Twin- fig. a

Makuchowski twin is a dendroid where each nonconstant open mapping is light. See [60, Example 2.1].

Chapter 5

Fans

5.1 Harmonic Fan

We define the *Harmonic Fan* $F_H = \cup\{ve : e \in H\}$, where ve is the segment joining $v = (0,1)$ with $e \in H$, $H = \{0\} \cup \{1/n : n \in N\}$. We consider F_H as a subset of the Euclidean plane with the induced topology.

See [39, p.9].

5.2 Harmonic Shredded Fan

To define the Harmonic Shredded Fan F_{HS} we cut the 'arms' ve in the Harmonic Fan F_H in such a way, that their lengths run through all rationals in the unit interval (we do not cut the vertical segment through the origin). We notice that all Harmonic shredded fans are homeomorphic.

See [39, p.31].

5.3 Cantor Fan

We define the *Cantor Fan* $F_C = \cup\{ve : e \in C\}$, where ve is the segment joining $v = (1/2, 1)$ with all points e in the Cantor Ternary Set C. We consider F_C as a subset of the Euclidean plane with the induced topology.

See [39, p.9].

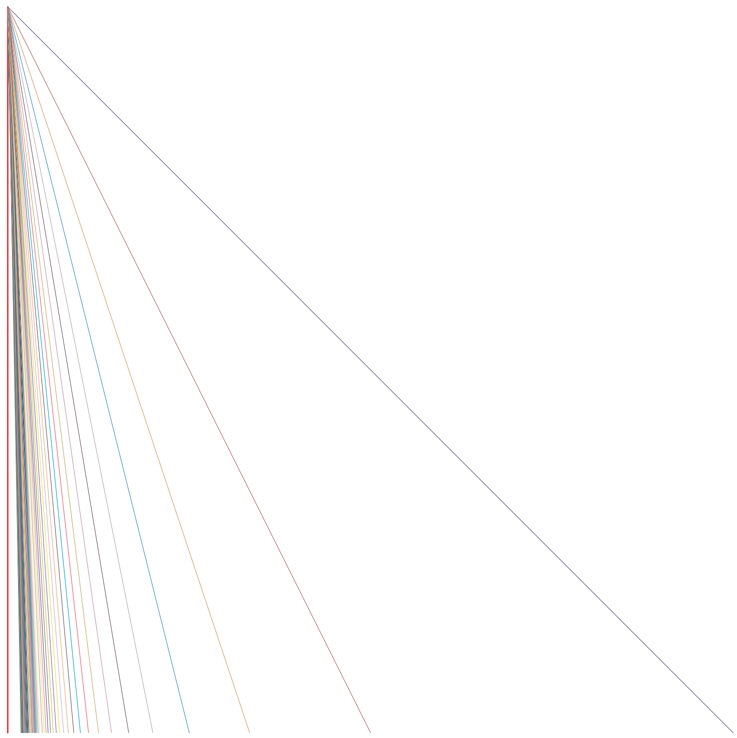

Figure 5.1: Harmonic Fan- fig. a

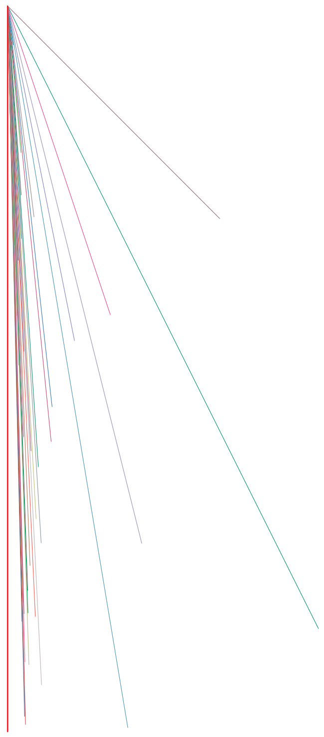

Figure 5.2: Harmonic Shredded Fan- fig. a

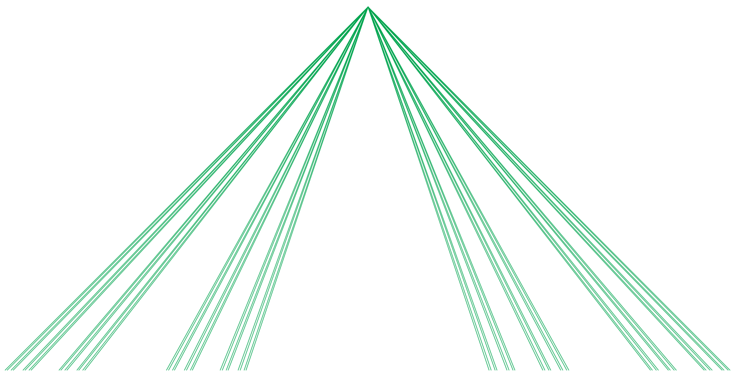

Figure 5.3: Cantor Fan- fig. a

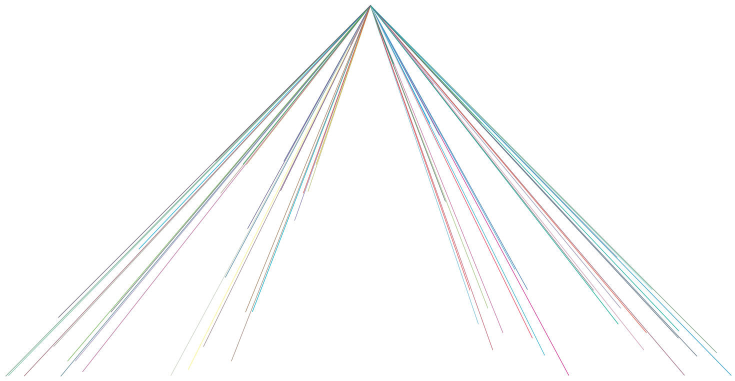

Figure 5.4: Lelek Fan- fig. a

5.4 Lelek Fan

We define the *Lelek Fan* F_L a subfan of the Cantor Fan F_C such that the set $E(F_L)$ of its end points is dense in F_L, and $S(F_L) = \{v\} \cup E(F_L)$ is a connected set (where v denotes the top of the two fans F_C and $F_L \subset F_C$).

It is known, that any two Lelek Fans are homeomorphic. See [39, p.9] . Moreover the Lelek Fan is homeomorphic to all its confluent images (see [37,]).

Chapter 6

Planable continua

6.1 Makuchowski Umbrella

Makuchowski Umbrella is a hereditarily locally connected continuum where each nonconstant open mapping is light. This property disappears when we cut out those vertical segments due to the horizontal projection on appropriate vertical segment.

See [60, Example 2.5].

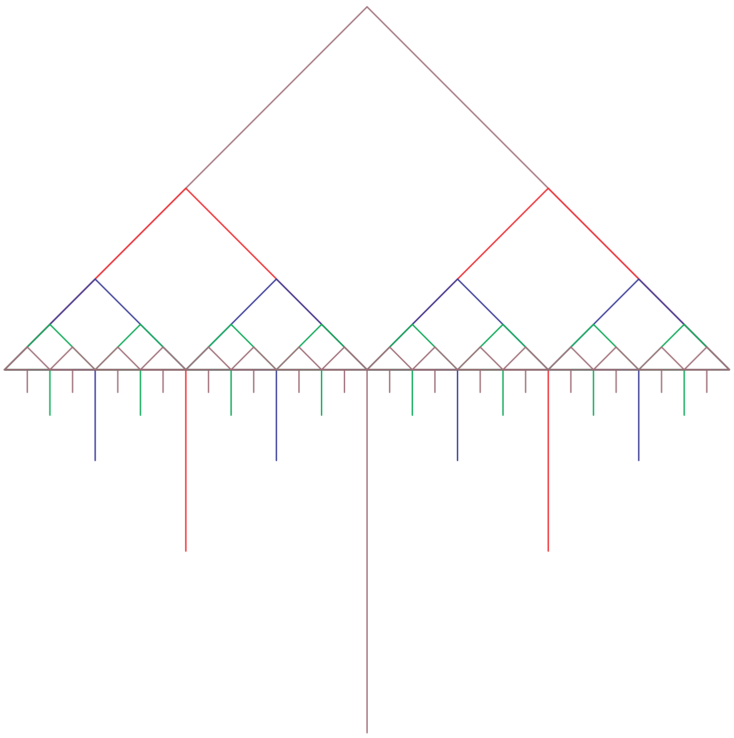

Figure 6.1: Makuchowski Umbrella- fig. a

Appendix A

Definitions

absolute end point

A point p of a continuum X is called an *absolute end point* of X if $X \setminus \{p\}$ is a composant of X.

absolute retract

An *absolute retract* is a space X such that if X is embedded as a closed subset X' of a space Y, then X' is a retract of Y.

absolutely terminal continuum

A proper subcontinuum K of a continuum X is said to be an *absolutely terminal continuum* of X provided that K is a terminal continuum of each subcontinuum L of X which properly contains K.

acyclic

A continuum X is said to be *acyclic* provided that each mapping from X into the unit circle $\mathbb{S}^1$ is homotopic to a constant mapping, i.e., for all mappings $f : X \to \mathbb{S}^1$ and $c : X \to \{p\} \subset \mathbb{S}^1$ there exists a mapping $h : X \times [0, 1] \to \mathbb{S}^1$ such that for each point $x \in X$ we have $h(x, 0) = f(x)$ and $h(x, 1) = c(x)$.

almost chainable

A continuum M is *almost chainable* if, for each positive number ε, there exists an open cover D of M with mesh ε and a chain C of elements of D and an end link L of C such that no element of $D \setminus C$ intersects any link of C other than L and every point of M is at a distance less than ε from some link of C.

aposyndetic

A connected space X is *aposyndetic at H* with respect to K if there is a closed connected subset of X with H in its interior and not intersecting K, and X is *aposyndetic* if it is aposyndetic at each point with respect to every other point.

A continuum X is said to be *aposyndetic* provided that for each point $p \in X$ and for each $q \in X \setminus \{p\}$ there exists a subcontinuum K of X and an open set U of X such that $p \in U \subset K \subset X \setminus \{q\}$ (see e.g.[65, Exercise 1.22, p. 12]).

arc

An *arc* is any space which is homeomorphic to the closed interval $[0, 1]$.

arc-smooth

Given a continuum X with an arc-structure A, the pair (X, A) (see arc-structure) is said to be *arc-smooth at a point $v \in X$* provided that the induced function $A_v : X \to C(X)$ defined by $A_v(x) = A(v, x)$ is continuous. Then the point v is called an *initial point* of (X, A). The pair (X, A) is said to be *arc-smooth* provided that there exists a point in X at which (X, A) is arc-smooth. An arbitrary space X is said to be *arc-smooth at a point $v \in X$* provided that there exists an arc-structure A on X for which (X, A) is arc-smooth at v. The space X is said to be *arc-smooth* if it is arc-smooth at some point (see [42, p. 546]). Note that a dendroid is smooth if and only if it is arc-smooth.

arc-structure

By an *arc-structure* on an arbitrary space X we understand a function $A : X \times X \to C(X)$ such that for every two distinct points x and y in X the set $A(x,y)$ is an arc from x to y and that the following metric-like axioms are satisfied for every points x, y and z in X:

(1) $A(x,x) = \{x\}$;

(2) $A(x,y) = A(y,x)$;

(3) $A(x,z) \subset A(x,y) \cup A(y,z)$,
with equality prevailing whenever $y \in A(x,z)$.

We put (X, A) to denote that the space X is equipped with an arc-structure A (see [42, p. 546]). Note that if there exists an arc-structure on a continuum, then the continuum is arcwise connected.

atomic

A surjective mapping $f : X \to Y$ between compact spaces is said to be (see [59, Chapter 3 and 4, p. 12-28]):
– *atomic* provided that for each subcontinuum K of X such that the set $f(K)$ is nondegenerate we have $K = f^{-1}(f(K))$;

atriodic

a continuum that contains no subcontinuum which is itself a triod is said to be *atriodic*

biconnected

A space is *biconnected* if it is not the sum of two mutually exclusive nondegenerate connected point sets.

bihomogeneous

Suppose X is a space such that, for each two points a and b, there is a homeomorphism of X onto itself such that $h(a) = b$ and $h(b) = a$. Then X is *bihomogeneous*.

Borel sets

The family F of *Borel sets* of a space S is the smallest family satisfying the conditions:

(i) Every closed set belongs to F;

(ii) If X is in F, then $S \setminus X$ is in F;

(iii) The countable intersection of elements of F belongs to F.

branch point

A *branch point* of a continuum is the vertex of a simple triod lying in that continuum.

chainable

A *simple chain* is a finite sequence $L_1, L_2, \cdots, L_n$ of open sets such that L_i intersects L_j if and only if $|i-j| \leq 1$. The terms of the sequence $L_1, L_2, \cdots, L_n$ are called the *links* of the chain. An ε*-chain* is a chain each of whose links has diameter less than ε. a continuum M is *chainable* if, for each positive number ε, M can be covered by an ε-chain.

chaotic

A nondegenerate topological space X is said to be *chaotic* if for any two distinct points p and q of X there exist an open neighborhood U of p and an open neighborhood V of q such that no open subset of U is homeomorphic to any open subset of V.

circle

A *circle* is any space which is homeomorphic to the unit circle.

completely regular

a continuum is *completely regular* if each nondegenerate subcontinuum has a nonempty interior.

composant

If a point p of a continuum X is given, then the *composant of* X *belonging to* p is defined to be the union of all proper subcontinua of X which contain p. A set is a *composant of* X provided it is the composant of X belonging to p for some point p of X.

confluent

A mapping f of a compact space X onto a compact space Y is *confluent* if, for each continuum C in Y, each component of $f^{-1}(C)$ is mapped onto C by f.

-connected

A topological space (X, τ) is said to be *σ-connected* provided that X is not the union of more than one and at most countably many nonempty, mutually disjoint, closed subsets.

A continuum is *δ-connected* if each two of its points can be connected by an hereditarily decomposable irreducible subcontinuum.

connected im kleinen

A space is *connected im kleinen* if it is aposyndetic at each point with respect to each closed point set not containing that point.

continuous selection of

Let a continuum X, a compact space Y and a function $F : X \to 2^Y$ be given. A function $f : X \to Y$ is called a *continuous selection of* F provided that it is continuous and $f(x) \in F(x)$ for each $x \in X$.

converges homeomorphically to the continuum

The statement that the sequence $M_1, M_2, \cdots$ *converges homeomorphically to the continuum* M means there exists a sequence $h_1, h_2, \cdots$ of homeomorphisms such that, for each positive integer i, h_i is a homeomorphism from M_i onto M and for each positive number ε there exists a positive integer N such that if $j > N$ then, for all x, $\mathrm{dist}(h_j(x), x) < \varepsilon$.

converge 0-regularly

A sequence of sets $\{A_n\}$ contained in a metric space X with a metric d is said to *converge* 0*-regularly* to its limit $A = \operatorname{Lim} A_n$ (see hyperspace) provided that for each $\varepsilon > 0$ there is a $\delta > 0$ and there is an index $n_0 \in \mathbb{N}$ such that if $n > n_0$ then for every two points $p, q \in A_n$ with $d(p, q) < \delta$ there is a connected set $C_n \subset A_n$ satisfying conditions $p, q \in C_n$ and $diam_n < \varepsilon$ (see [78, Chapter 9, §3, p. 174]).

convex

Given a continuum X with an arc-structure A, a subset Z of X is said to be *convex* provided that for each pair of points x and y of Z the arc $A(x, y)$ is a subset of Z. If Z is a convex subcontinuum of X, then $A|Z \times Z$ is an arc-structure on Z. We define X to be *locally convex at a point* $p \in X$ provided that for each open set U containing p there is a convex set Z such that $p \in \operatorname{int} Z \subset \operatorname{cl} Z \subset U$ (see [42, I.2, p. 548-549]).

continuum

A *continuum* is a nonempty, compact, connected metric space.

contractible

A continuum X is said to be *contractible* provided there is a homotopy $h : X \times [0,1] \to X$ such that for some point $p \in X$ we have $h(x,0) = x$ and $h(x,1) = p$ for each $x \in X$ (see e.g. [64, (16.2), p. 532]).

curve

A *curve* is any one-dimensional continuum.

cut point

Let $S, T)$ be a connected topological space, and let $p \in S$. If $S \setminus \{p\}$ is connected, then p is called a *non-cut point of* S. If $S \setminus \{p\}$ is not connected, then p is called a *cut point of* S.

C^*-smooth

Let X be a continuum. Define $C^* : C(X) \to C(C(X))$ by $C^*(A) = C(A)$. It is known that for any continuum X the function C^* is upper semi-continuous, [64, Theorem 15.2, p. 514], and it is continuous on a dense G_δ subset of $C(X)$, [64, Corollary 15.3, p. 515]. A continuum X is said to be *C^*-smooth at* $A \in C(X)$ provided that the function C^* is continuous at A. A continuum X is said to be *C^*-smooth* provided that the function C^* is continuous on $C(X)$, i.e., at each $A \in C(X)$ (see [64, Definition 5.15, p. 517]).

cyclic element

Let a continuum X be locally connected and let $p \in X$. By a *cyclic element* of X containing p we mean either the singleton $\{p\}$ if p is a cut point or an end point of X, or the set consisting of p and of all points $x \in X$ such that no point of X cuts X between p and x, otherwise (see e.g. [78, Chapter 4, §2, p. 66]).

decomposable

a continuum X is said to be *decomposable* provided that X can be written as the union of two proper subcontinua.

deformation retract of

Let X be a space and let A and B be subspaces of X with $A \subset B$. Then A is called a *deformation retract of* B over X provided that the identity mapping $i_B : B \to B$ is homotopic in X to a retraction $r : B \to A$. Further, A is called a *strong deformation retract of* B over X provided that it is a deformation retract of B over X and the homotopy keeps the points of A fixed throughout the entire deformation of B into A (see e.g. [41, Definition 6.3, p. 324]).

dendroid

A *dendroid* is an arcwise connected, hereditarily unicoherent continuum.

dendrite

A *dendrite* means a locally connected continuum containing no simple closed curve.

dispersion point

A point p of a nondegenerate connected space X is an *explosion point*, or *dispersion point*, of X provided that $X \setminus \{p\}$ is totally disconnected.

end point

Point of order 1 in a continuum X is called *end point* of X; the set of all end points of X is denoted by $E(X)$.

equivalent

Let $\mathfrak{M}$ be a class of mappings. Two spaces X and Y are said to be *equivalent with respect to* $\mathfrak{M}$ (shortly $\mathfrak{M}$-*equivalent*) if there are two mappings, both in $\mathfrak{M}$, one from X onto Y and the other from Y onto X. If $\mathfrak{M}$ means the class of monotone mappings, we say that X and Y are *monotonely equivalent*.

F-equivalent

If X and Y are continua, we say that Y is F-*equivalent* to X provided there is a mapping in F from X onto Y and a mapping in F from Y onto X.

feebly monotone

Let X and Y be continua. A mapping $f : X \to Y$ is said to be *feebly monotone* provided that if A and B are proper subcontinua of Y such that $Y = A \cup B$, then their inverse images $f^{-1}(A)$ and $f^{-1}(B)$ are connected.

finitely Suslinean

a continuum X is *finitely Suslinean* if, for each positive number ε, X does not contain infinitely many mutually exclusive subcontinua of diameter greater than ε.

finitely linear

A function $f\, I \to Y$, where I is a closed interval of the real line, is called *finitely linear* provided there exists a positive integer m such that I can be decomposed, for each $\varepsilon > 0$, into a finite number of closed subintervals

$I_1, I_2, \cdots, I_k$ each of length less than ε and with the property that the set $f(I_i)$ meets at most m of the sets $f(I_1), f(I_2), \cdots, f(I_k)$ for $i = 1, 2, \cdots, k$.

fixed point

We say that a function $F : X \to 2^X$ (or a function $F : X \to C(X)$) *has a fixed point* provided that there is a point $x \in X$ such that $x \in F(x)$.

fixed set property

A topological space X is said to have the *fixed set property* for a certain class C of maps of X onto itself provided there exists, for each non-empty closed set A in X, a map f in C such that $f(x) = x$ if and only if x is in A.

fixed point self-homeomorphic

A topological space X is called *fixed point self-homeomorphic* if for any point p, any neighborhood U of p, there is an embedding $h : X \to U$ with $h(p) = p$ and $p \in \operatorname{int} h(X)$.

fixed ball self-homeomorphic

A topological space X is called *fixed ball self-homeomorphic* if for any point p, any neighborhood U of p, there is a neighborhood V of p with $V \subseteq U$ and an embedding $h : X \to U$ satisfying $h|_V = \mathrm{id}_V$.

free arc

A *free arc* in X is any subset of X of the form $A \setminus \{x, y\}$ where A is an arc in X with end points x and y and $A \setminus \{x, y\}$ is open in X.

graph

A *graph* is a continuum which can be written as the union of finely many arcs any two of which are either disjoint or intersect only in one or both their end points.

half-ray curve

a continuum is a *half-ray curve* if it is a continuous 1-1 image of the nonnegative reals.

hereditary

A property of a continuum X is said to be *hereditary* if every subcontinuum of X has the property. In particular, a continuum is said to be *hereditarily unicoherent* if the intersection of any two of its subcontinua is connected.

hereditarily

Let $\mathfrak{M}$ be a class of mappings between continua. A mapping $f : X \to Y$ between continua is said to be *hereditarily* $\mathfrak{M}$ provided that its restriction to any subcontinuum of the domain X is in $\mathfrak{M}$ (see [59, Chapter 4, Section B, p. 16]).

hereditarily decomposable

a continuum X is said to be *hereditarily decomposable* provided that each nondegenerate subcontinuum of X (as well as X itself) is decomposable.

hereditarily equivalent

a continuum is *hereditarily equivalent* if it is homeomorphic to each of its nondegenerate subcontinua.

hereditarily indecomposable

a continuum X is said to be *hereditarily indecomposable* provided that each of its subcontinua is indecomposable, that is, for each subcontinuum $C \subset X$ and for every continua A and B such that $A \cup B = C$ we have either $A = C$ or $B = C$.

hereditarily unicoherent

A connected topological space is said to be *hereditarily unicoherent* provided that each of its closed, connected subsets is unicoherent.

homogeneous

Let $\mathfrak{M}$ be a class of mappings. A space X is said to be *homogeneous with respect to* $\mathfrak{M}$ (or shortly $\mathfrak{M}$-*homogeneous*) provided that for every two points p and q of X there is a surjective mapping $f : X \to X$ such that $f(p) = q$ and $f \in \mathfrak{M}$. If $\mathfrak{M}$ is the class of homeomorphisms, we get the concept of a *homogeneous* space.

HU-terminal

A subcontinuum K of a hereditarily unicoherent continuum X is said to be a *HU-terminal continuum* of X provided that K is contained in an irreducible subcontinuum of X and for every irreducible subcontinuum I of X containing K there is a point $x \in X$ such that I is irreducible about the union $K \cup \{x\}$.

hyperspace

Given a continuum X with a metric d, we let 2^X to denote the *hyperspace* of all nonempty closed subsets of X equipped with the *Hausdorff metric* H defined by

$$H(A,B) = \max\{\sup\{d(a,B) : a \in A\}, \sup\{d(b,A) : b \in B\}\} \quad \text{for } A, B \in 2^X$$

(see e.g. [64, (0.1), p. 1 and (0.12), p. 10]). If $H(A, A_n)$ tends to zero as n tends to infinity, we put $A = \operatorname{Lim} A_n$. Further, we denote by $F_1(X)$ the hyperspace of singletons of X, and by $C(X)$ the hyperspace of all subcontinua of X, i.e., of all connected elements of 2^X. Since X is homeomorphic to $F_1(X)$, there is a natural embedding of X into $C(X)$, and so we can write $X \subset C(X) \subset 2^X$. Thus one can consider a retraction from either $C(X)$ or 2^X onto X.

indecomposable

a continuum X is said to be *indecomposable* provided that X cannot be written as the union of two proper subcontinua.

induced mapping

Let $f : X \to Y$ be a mapping between continua. Then the *induced mapping* $C(f) : C(X) \to C(Y)$ is defined by $C(f)(A) = f(A)$, where A in the left member of the equality means an element of $C(X)$, while in the right one it is understood as a subcontinuum of X (see [64, (0.49), p. 23]).

inverse limit

The reader can find necessary information on the inverse limits of inverse sequences e.g. in Section 2 of the second chapter of [65], as well as in [55].

irreducible

Let X be a continuum and $A \subset X$. Then, X is said to be *irreducible about* A provided no proper subcontinuum of X contains A. a continuum X is said to be *irreducible* provided that X is irreducible about $\{p, q\}$ for some $p, q \in X$.

light

A mapping is *light* if each point inverse is totally disconnected.

like

If X is a metric space, a mapping f from X to a space Y is an *ε-map* if, for each point y of Y, $diam f^{-1}(y)) \leq \varepsilon$. If C is a collection of continua, a continuum M is *C-like* if, for every positive number ε, there exists an ε-map of M onto an element of C. In particular, a continuum is *tree-like* if, for some collection C of trees, M is C-like.

A concept of a tree-like continuum can be defined in several (equivalent) ways. One of them is the following. A continuum X is said to be *tree-like* provided that for each $\varepsilon > 0$ there is a tree T and a surjective mapping $f : X \to T$ such that f is an ε-mapping (i.e., $diam^{-1}(y) < \varepsilon$ for each $y \in T$). Let us mention that a continuum X is tree-like if and only if it is the inverse limit of an inverse sequence of trees with surjective bonding mappings. Compare e.g. [65, p. 24].

Using a concept of a nerve of a covering, one can reformulate the above definition saying that a continuum X is be tree-like provided that for each $\varepsilon > 0$ there is an ε-covering of X whose nerve is a tree.

Finally, the original definition using tree-chains can be found e.g. in Bing's paper [4, p. 653].

locally

Let $\mathfrak{M}$ be a class of mappings between compact spaces. A surjective mapping $f : X \to Y$ between continua is said to be *locally* $\mathfrak{M}$ provided that for each point $x \in X$ there is a closed neighborhood V of x such that $f(V)$ is is a closed neighborhood of $f(x)$ and that the restriction $f|V$ is in $\mathfrak{M}$ (see [59, Chapter 4, Section C, p. 18]).

locally confluent

A mapping f from a space X onto a space Y is *locally confluent* if, for each point y of Y, there is an open set U containing y such that $f^{-1}(\overline{U})$ is confluent.

local homeomorphism

A surjective mapping $f : X \to Y$ between compact spaces is said to be (see [59, Chapter 3 and 4, p. 12-28]):
– a *local homeomorphism* provided that for each point $x \in X$ there exists a an open neighborhood U of x such that $f(U)$ is an open neighborhood of $f(x)$ and that f restricted to U is a homeomorphism between U and $f(U)$.

local separating point

A point p of a locally compact separable metric space L is a *local separating point* of L provided there exists an open set U of L containing p and two points x and y of the component containing p of U such that $U \setminus \{p\}$ is the sum of two mutually separated point sets, one containing x and the other containing y.

MO-mapping

A surjective mapping $f : X \to Y$ between compact spaces is said to be (see [59, Chapter 3 and 4, p. 12-28]):
– an *MO-mapping* provided that it can be represented as the composition of two mappings, $f = f_2 \circ f_1$ such that f_1 is open and f_2 is monotone.

monotone

A continuous function $f : X \to Y$ is said to be *monotone* provided $f^{-1}(y)$ is connected for each $y \in Y$.

mutually aposyndetic

a continuum M is *mutually aposyndetic* if, for each two points A and B of M, there exist mutually exclusive subcontinua H and K of M containing A and B, respectively, in their interiors. a continuum is *strictly non-mutually aposyndetic* if each two of its subcontinua with interiors intersect.

natural projection

Let continua X and Y be given. A mapping $f : X \times Y \to X$ is called the *natural projection* provided that it is defined by $f((x, y)) = x$.

neat

A dendroid X is said to be *neat* provided that each one of its subdendroids has no improper shore point (see [71, p. 939]).

Let $\mathfrak{M}$ be a class of mappings. A class $\mathfrak{M}$ of mappings is said to be *neat* provided that if all homeomorphisms are in $\mathfrak{M}$ and the composition of any two mappings in $\mathfrak{M}$ is also in $\mathfrak{M}$.

near homeomorphism

A mapping $f : X \to Y$ is called a *near homeomorphism* provided that for each $\varepsilon > 0$ there is a homeomorphism $h : X \to Y$ such that $\sup\{d(f(x), h(x)) : x \in X\} < \varepsilon$.

OM-mapping

A surjective mapping $f : X \to Y$ between compact spaces is said to be (see [59, Chapter 3 and 4, p. 12-28]):
– an *OM-mapping* provided that it can be represented as the composition of two mappings, $f = f_2 \circ f_1$ such that f_1 is monotone and f_2 is open.

open

A mapping f of a space X into a space Y is *open* if, for each open A in X the image $f(A)$ is open in Y.

openly minimal

We say that a dendrite X is *openly minimal* provided that every open image of X can be openly mapped onto X.

orbit

Given a space X let $\mathcal{H}(X)$ stand for the group of autohomeomorphisms of X. If a point $p \in X$ is fixed, then $\{h(p) \in X : h \in \mathcal{H}(X)\}$ is called an *orbit of* p. Orbits of points of X either are mutually disjoint or coincide, and their union is the whole X.

order

Let (X, T) be a topological space, and let $A \subset X$. Let β be a cardinal number. We say that A *is of order less than equal to* β in X, written $\mathrm{ord}(A, X) \leq \beta$, provided that for each $U \in T$ such that $A \subset U$, there exists $V \in T$ such that $A \subset V \subset U$ and $|Bd(V)| \leq \beta$.

We say that A *is of order* β *in* X, written $\mathrm{ord}(A, X) = \beta$, provided that $\mathrm{ord}(A, X) \leq \beta$ and $\mathrm{ord}(A, X) \nleq \alpha$ for any cardinal number $\alpha < \beta$.

A concept of an *order* of a point p in a continuum X (in the sense of Menger-Urysohn), written $\mathrm{ord}\,(p, X)$, is defined as follows. Let $\mathfrak{n}$ stand for a cardinal number. We write:

$\mathrm{ord}\,(p, X) \leq \mathfrak{n}$ provided that for every $\varepsilon > 0$ there is an open neighborhood U of p such that *diam* $\leq \varepsilon$ and $\mathrm{card}\,\mathrm{bd}\, U \leq \mathfrak{n}$;

$\mathrm{ord}\,(p, X) = \mathfrak{n}$ provided that $\mathrm{ord}\,(p, X) \leq \mathfrak{n}$ and for each cardinal number $\mathfrak{m} < \mathfrak{n}$ the condition $\mathrm{ord}\,(p, X) \leq \mathfrak{m}$ does not hold;

$\mathrm{ord}\,(p, X) = \omega$ provided that the point p has arbitrarily small open neighborhoods U with finite boundaries $\mathrm{bd}\, U$ and $\mathrm{card}\,\mathrm{bd}\, U$ is not bounded by any $n \in \mathbb{N}$.

Thus, for any continuum X we have

$$\mathrm{ord}\,(p, X) \in \{1, 2, \ldots, n, \ldots, \omega, \aleph_0, 2^{\aleph_0}\}$$

(convention: $\omega < \aleph_0$); see [55, §51, I, p. 274].

Let a dendroid X and a point $p \in X$ be given. Then p is said to be a *point of order at least* $\mathfrak{m}$ *in the classical sense* provided that p is the center of an $\mathfrak{m}$-od contained in X. We say that p is a *point of order* $\mathfrak{m}$ *in the classical sense* provided that $\mathfrak{m}$ is the minimum cardinality for which the above condition is satisfied (see [8, p. 229]).

order preserving mapping

For each point p of a continuum X equipped with an arc-structure $A : X \times X \to C(X)$ we define a partial order $\leq_p$ by letting $x \leq_p y$ whenever $A(p,x) \subset A(p,y)$. Let X and Y be continua with fixed arc-structures A and B, respectively. We say that a surjective mapping $f : X \to Y$ is a *$\leq_p$-mapping* provided that $x \leq_p y$ in X implies that $f(x) \leq_{f(p)} f(y)$ in Y. If, in addition, $Y \subset X$, $B = A|(Y \times Y)$, and f is a retraction, then f is called a *$\leq_p$-retraction* (or *$\leq_p$-preserving retraction*). The concept of a *$<_p$-mapping* is defined in a similar manner (with $f(x) \neq f(y)$ implied by $x \neq y$). For *order preserving mapping*s see e.g. [42, I.7, p. 553].

ordinary point

Point of order 2 is called *ordinary point* of X; the set of all ordinary points of X is denoted by $O(X)$.

periodic point of

Let a continuum X and a mapping $f : X \to X$ be given. For each natural number n denote by f^n the n-th iteration of f. A point $p \in X$ is called a *periodic point of* f provided that there is $n \in \mathbb{N}$ such that $f^n(p) = p$ The set of periodic points of a mapping $f : X \to X$ are denoted by $P(f)$.

periodic-recurrent property

The set of periodic points and of recurrent points of a mapping $f : X \to X$ are denoted by $P(f)$ and $R(f)$ respectively. Clearly $P(f) \subset R(f)$. A continuum X is said to have the *periodic-recurrent property* (shortly *PR-property*) provided that for every mapping $f : X \to X$ the equality $\operatorname{cl} P(f) = \operatorname{cl} R(f)$ holds.

pointwise self-homeomorphic

A topological space X is called *pointwise self-homeomorphic at a point* $x \in X$ if for any neighborhood U of x there is a set V such that $x \in V \subseteq U$ and V

is homeomorphic to X. The space X is called *pointwise self-homeomorphic* if it is pointwise self-homeomorphic at each of its points.

pseudo-arc

A *pseudo-arc* is the only (up to homeomorphism) hereditarily indecomposable arc-like continuum.

pseudo-confluent

A mapping from X to Y is *pseudo-confluent* if every irreducible continuum in Y is the image of a continuum in X.

property of Kelley

A continuum X is said to have the *property of Kelley* provided that for each point $x \in X$, for each subcontinuum K of X containing x and for each sequence of points x_n converging to x there exists a sequence of subcontinua K_n of X containing x_n and converging to the continuum K (see e.g. [64, Definition 16.10, p. 538]).

quasi-monotone

A surjective mapping $f : X \to Y$ between compact spaces is said to be (see [59, Chapter 3 and 4, p. 12-28]) *quasi-monotone* provided that for each subcontinuum Q of Y with the nonempty interior the set $f^{-1}(Q)$ has a finite number of components and f maps each of them onto Q.

radially convex at

Given a continuum X with an arc-structure A and a point $p \in X$, a metric d on X is said to be *radially convex at* p provided that $d(p, z) = d(p, y) + d(y, z)$ for every points $y, z \in X$ with $y \in A(p, z)$ (see [42, I.4, p. 551]).

ramification point

Point of order at least 3 is called *ramification point* of X; the set of all ramification points of X is denoted by $R(X)$.

rational

A continuum is *rational* if every two of its points can be separated by a countable point set.

real curve

A continuum is a *real curve* if it is a continuous 1-1 image of the real line.

recurrent point of

Let a continuum X and a mapping $f : X \to X$ be given. For each natural number n denote by f^n the n-th iteration of f. A point $p \in X$ is called a *recurrent point of* f provided that for every neighborhood U of p there is $n \in \mathbb{N}$ such that $f^n(p) \in U$. The set of recurrent points of a mapping $f : X \to X$ are denoted by $R(f)$.

refinable (monotonely)

A map r from a compact metric space X onto a compact metric space Y is *(monotonely) refinable* if, for each positive number ε, there is a (monotone) ε-map f from X onto Y such that, for each x in X,

$$d\left(f\left(x\right), r\left(x\right)\right) < \varepsilon.$$

regular

A continuum is *regular* if each two of its points can be separated by a finite point set.

retract, retraction

Let X and Y be continua. A mapping $f : X \to Y$ is said to be a *retraction* provided that $Y \subset X$ and the restriction $f|Y : Y \to f(Y) \subset X$ is the identity; then Y is called a *retract* of X.

rigid

A nondegenerate topological space X is said to be *rigid* if it has a trivial autohomeomorphism group, i.e., if the only homeomorphism of X onto X is the identity.

selectible

A continuum X is said to be *selectible* provided that there exists a mapping $\sigma : C(X) \to X$ (called a *selection* for $C(X)$) such that $\sigma(A) \in A$ for each continuum $A \subset X$ (see e.g. [64, p. 253]).

scattered

A point set X is *scattered* if every subset Y of X has a point that is not a limit point of Y.

self-homeomorphic

A topological space X is called *self-homeomorphic* if for any open set $U \subseteq X$ there is a set $V \subseteq U$ such that V is homeomorphic to X.

semi-aposyndetic

A space X is *semi-aposyndetic* if, for each two of its points, X is aposyndetic at one of them with respect to the other.

semi-confluent

A surjective mapping $f : X \to Y$ between compact spaces is said to be (see [59, Chapter 3 and 4, p. 12-28]):
– *semi-confluent* provided that for each subcontinuum Q of Y and for every two components C_1 and C_2 of $f^{-1}(Q)$ either $f(C_1) \subset f(C_2)$ or $f(C_2) \subset f(C_1)$.

semi-continuum

A *semi-continuum* is a continuumwise connected point set.

semi-continuous

Let a continuum X, a compact space Y and a function $F : X \to 2^Y$ be given. Put

$$F^{-1}(B) = \{x \in X : F(x) \cap B \neq \emptyset\}.$$

The function F is said to be *lower (upper) semi-continuous* provided that $F^{-1}(B)$ is open (closed) for each open (closed) subset $B \subset Y$. It is said to be *continuous* provided that it is both lower and upper semi-continuous. This notion of continuity agrees with the one for mappings between metric spaces.

semi-locally connected

A continuum M is *semi-locally connected* at the point p if, for every domain D containing p, there exits a domain E lying in D and containing p such that $M \setminus E$ has only a finite number of components. A continuum is *totally non-semi-locally connected* if it is not semi-locally connected at any point.

simple closed curve

A *simple closed curve* is any space which is homeomorphic to the unit circle.

simple $\mathfrak{m}$-od

Let $\mathfrak{m}$ be a cardinal number. By a *simple* $\mathfrak{m}$*-od* with the center p we mean the union of $\mathfrak{m}$ arcs every two of which have p as the only common point.

simple triod

The union of three arcs emanating from a point v is called a *simple triod* provided the singleton v is the intersection of any two of the arcs ; the point v is then called the *top* of the simple triod, and the arcs are called its *arms*.

shore point

A point p of a dendroid X is called a *shore point* of X if there exists a sequence of subdendroids X_n of $X \setminus \{p\}$ such that $X = \operatorname{Lim} X_n$. A shore point of X that is not an end point of X is called an *improper shore point* of X (see [71, p. 939]).

smooth (dendroid)

A dendroid X is said to be *smooth at a point* $p \in X$ provided that for each point $x \in X$ and for each sequence of points $\{x_n\}$ in X tending to x, the sequence of arcs px_n tends to the arc px. A dendroid X is said to be *smooth* provided that it is smooth at some point $p \in X$ (see [34, p. 298]).

strongly chaotic

A nondegenerate topological space X is said to be *strongly chaotic* if for any two distinct points p and q of X there exist open neighborhoods U of p and V of q respectively such that no open subset of U is homeomorphic to any subset of V;

strongly pointwise self-homeomorphic

A topological space X is called *strongly pointwise self-homeomorphic at a point* $x \in X$ if for any neighborhood U of x there is a neighborhood V of x such that $x \in V \subseteq U$ and V is homeomorphic to X. The space X is called *strongly pointwise self-homeomorphic* if it is strongly pointwise self-homeomorphic at each of its points.

strongly rigid

A nondegenerate topological space X is said to be *strongly rigid* if the only homeomorphism of X into X is the identity of X onto itself.

strongly self-homeomorphic

A topological space X is called *strongly self-homeomorphic* if for any open set $U \subseteq X$ there is a set $V \subseteq U$ with nonempty interior such that V is homeomorphic to X.

strongly unicoherent1

A continuum X is *strongly unicoherent* provided X is unicoherent and each proper subcontinuum with interior is unicoherent.

strongly unicoherent2

A continuum X is said to be *strongly unicoherent2* provided that it is unicoherent and for each pair of its proper subcontinua A and B such that $X = A \cup B$, each of A and B is unicoherent (see [2, p. 587]).

Suslinean

A continuum is *Suslinean* if it does not contain uncountably many mutually exclusive nondegenerate subcontinua.

$T_A(x)$

For each point x of a continuum X with an arc-structure A we define (see [42, I.3, p. 550]) $T_A(x) = \{y \in X :$ each convex subcontinuum of X with y in its interior contains $x\}$. Since $T_A(x)$ is always closed, we have $T_A : X \to 2^X$.

terminal continuum

A proper subcontinuum K of a continuum X is said to be a *terminal continuum* of X provided that if whenever A and B are proper subcontinua of X having union equal to X such that $A \cap K \neq \emptyset \neq B \cap K$, then either $X = A \cup K$ or $X = B \cup K$.

thin

A topological space S is *thin* if and only if for each two homeomorphic subset A and B of S, there is a homeomorphism h of S onto itself such that $h(A) = B$.

translation

If P and Q are subspaces of a metric space X with a metric d, and ε is a positive number, then a mapping $g : P \to Q$ is called an *ε-translation* provided that $d(p, g(p)) < \varepsilon$ for each point $p \in P$.

tree

A *tree* is a graph which contains no simple closed curve. A tree as a one-dimensional acyclic connected polyhedron, i.e., a dendrite with finitely many end points.

triod

A continuum is called a *triod* provided it contains a subcontinuum whose complement is the union of three nonempty pairwise disjoint open sets.

unicoherent

A connected topological space S is said to be *unicoherent* provided that whenever A and B are closed, connected subsets of S such that $S = A \cup B$, then $A \cap B$ is connected.

Let a continuum X and its subcontinuum Y be given. Then X is said to be *unicoherent at* Y provided that for each pair of proper subcontinua A and B of X such that $X = A \cup B$ the intersection $A \cap B \cap Y$ is connected (see [72, p. 146]).

uniquely arcwise connected

A continuum is said to be *uniquely arcwise connected* provided that for every two of its points there is exactly one arc in the continuum joining these points.

universal

Let a class $\mathcal{S}$ of spaces be given. A member U of $\mathcal{S}$ is said to be *universal for* $\mathcal{S}$ if every member of $\mathcal{S}$ can be embedded in U, i.e., if for every $X \in \mathcal{S}$ there exists a homeomorphism $h : X \to h(X) \subset U$. Accordingly, a dendrite is said to be *universal* if it contains a homeomorphic image of any other dendrite.

weak cut point

A point x of a continuum M is a *weak cut point* of M if there are two points p and q of M such that every subcontinuum of M that contains both p and q also contains x.

weakly chainable

A continuum is *weakly chainable* if it is a continuous image of a chainable continuum.

weakly confluent

A surjective mapping $f : X \to Y$ between compact spaces is said to be (see [59, Chapter 3 and 4, p. 12-28]) *weakly confluent* provided that for each subcontinuum Q of Y there is a component the set $f^{-1}(Q)$ which is mapped under f onto Q.

weakly hereditarily unicoherent

A continuum is said to be *weakly hereditarily unicoherent* if the intersection of any two of its subcontinua with nonempty interiors is connected (see [72, p. 152] and references therein).

weakly monotone

A surjective mapping $f : X \to Y$ between compact spaces is said to be (see [59, Chapter 3 and 4, p. 12-28]) *weakly monotone* provided that for each subcontinuum Q of Y with the nonempty interior each component the set $f^{-1}(Q)$ is mapped under f onto Q.

weakly smooth at a point

A dendroid X is said to be *weakly smooth at a point* $p \in X$ provided that the subspace of 2^X consisting of all subarcs of X of the form px for $x \in X$ is compact (see [57, p. 113]).

weakly Suslinian

A continuum X is said to be *weakly Suslinian* provided that X is not the union of more than one pairwise disjoint nondegenerate subcontinua of X.

widely connected

A nondegenerate connected space X is *widely connected* if each nondegenerate connected subset of X is dense in X.

width $w(X)$

For any compact metric space X, the *width* $w(X)$ of X is the l.u.b. of the set of all real numbers a which satisfy the following condition: for each $\varepsilon > 0$, there exists a finite open cover C of X such that $meshC) < \varepsilon$ and for each chain C' which is a subcollection of C there is a member A of C such that $d(A, C'^*) \geq a$.

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Bibliography

[1] D. Arévalo, W. J. Charatonik, P. Pellicer Covarrubias and L. Simón, *On dendrites with the closed set of end points*, in preparation.

[2] D. E. Bennett, *Aposyndetic properties of unicoherent continua*, Pacific J. Math. **37** (1971), 585–589.

[3] E. S. Berney, *A chaotic space*, Acta Math. Acad. Sci. Hungar. **25** (1974), 1–4.

[4] R. H. Bing, *Snake-like continua*, Duke Math. J. **18** (1951), 653–663.

[5] K. Borsuk, Theory of retracts, PWN, Warszawa 1967.

[6] W. D. Bula and L. G. Oversteegen, *A characterization of smooth Cantor bouquets*, Proc. Amer. Math. Soc. **108** (1990), 529–534.

[7] C. E. Burgess, *Continua and various types of homogeneity*, Trans. Amer. Math. Soc. **88** (1958), 366–374.

[8] J. J. Charatonik, *On ramification points in the classical sense*, Fund. Math.**51** (1962), 229–252.

[9] J. J. Charatonik, *The fixed point property for set-valued mappings of some acyclic curves*, Colloq. Math. **26** (1972), 331–338.

[10] J. J. Charatonik, *Problems and remarks on contractibility of curves*, General Topology and its Relations to Modern Analysis and Algebra IV, Proc. Fourth Prague Topological Symposium, 1976; Part B Contributed Papers, Society of Czechoslovak Mathematicians and Physicists, Prague 1977, pp. 72–76.

[11] J. J. Charatonik, *A chaotic dendrite*, Bull. Acad. Polon. Sci. Sér. Sci. Math. Astronom. Phys. **25** (1977), 1261–1263.

[12] J. J. Charatonik, *On chaotic curves*, Colloq. Math. **41** (1979), 219–236.

[13] J. J. Charatonik, *Open mappings of universal dendrites*, Bull. Acad. Polon. Sci. Sér. Sci. Math. **28** (1980), 489–494.

[14] J. J. Charatonik, *Smooth dendroids without ordinary points*, Fund. Math. **122** (1984), 61–70.

[15] J. J. Charatonik, *Generalized homogeneity of curves and a question of H. Kato*, Bull. Polish Acad. Sci. Math. **36** (1988), 409–411.

[16] J. J. Charatonik, *Dendroids without ordinary points*, Rend. Circ. Mat. Palermo (2) Suppl. **24** (1990), 321-326.

[17] J. J. Charatonik, *Monotone mappings of universal dendrites*, Topology Appl. **38** (1991), 163–187.

[18] J. J. Charatonik, *On acyclic curves. A survey of results and problems*, Bol. Soc. Mat. Mexicana (3) **1** (1995), 1–39.

[19] J. J. Charatonik, *Homeomorphisms of universal dendrites*, Rend. Circ. Mat. Palermo (2) **44** (1995), 457–468.

[20] J. J. Charatonik, *An uncountable family of monotone homogeneous dendrites*, Glasnik Mat. **31 (51)** (1996), 289–294.

[21] J. J. Charatonik, *On sets of periodic and of recurrent points*, Publ. Inst. Math. (Beograd) (N. S.), to appear.

[22] J. J. Charatonik, *On chaotic dendrites*, preprint.

[23] J. J. Charatonik, *On generalized rigidity*, preprint.

[24] J. J. Charatonik, *On feebly monotone and related classes of mappings*, preprint

[25] J.J. Charatonik ... ?

[26] J.J. Charatonik, *On acyclic curves, a survey of results and problems* , Bol.Soc.Mat. Mexicana , **3,2** (1995), 1 - 39 .

[27] J. J. Charatonik and W. J. Charatonik, *Connected subsets of dendrites and separators of the plane*, Topology Appl. **36** (1990), 233–245.

[28] J. J. Charatonik and W. J. Charatonik, *Strongly chaotic dendrites*, Colloq. Math. **70** (1996), 181–190.

[29] J. J. Charatonik and W. J. Charatonik, *Monotone homogeneity of dendrites*, Comment. Math. Univ. Carolin. **38** (1997), 361–370.

[30] J. J. Charatonik and W. J. Charatonik, *Periodic-recurrent property of some continua*, Bull. Austral. Math. Soc.**56** (1997), 109–118.

[31] J. J. Charatonik, W. J. Charatonik and A. Illanes, *Openness of induced mappings*, Topology Appl., to appear.

[32] J. J. Charatonik, W. J. Charatonik and P. Krupski, *Dendrites and light open mappings*, Proc. Amer. Math. Soc., to appear.

[33] J. J. Charatonik, W. J. Charatonik and J. R. Prajs, *Mapping hierarchy for dendrites*, Dissertationes Math. (Rozprawy Mat.) **333** (1994), 1–52.

[34] J. J. Charatonik and C. Eberhart, *On smooth dendroids*, Fund. Math. **67** (1970), 297–322.

[35] W. J. Charatonik, *The Lelek fan is unique*, Houston J. Math. **15** (1989), 27–34.

[36] W. J. Charatonik and A. Dilks, *On self-homeomorphic spaces*, Topology Appl. **55** (1994), 215–238.

[37] W.J. Charatonik, *The Lelek fan is unique* , Houston J.Math. , **15** (1989), 27 - 34 .

[38] J.J. Charatonik, W.J. Charatonik, *Monotone homogeneity of dendrites* , Comment. Math. Univ. Carolinae , **38,2** (1997), 361 - 370 .

[39] J.J. Charatonik, W.J. Charatonik, S. Miklos, *Confluent mappings of fans, Dissertationes Mathematicae*, **301** (1990), 1 - 86 .

[40] J. J. Charatonik, P. Pyrih : *Continua with light open mappings*, Questiones and Answers in General Topology **17,2** (1999).

[41] J. Dugundji, Topology, Allyn and Bacon, Boston, 1966.

[42] J. B. Fugate, G. R. Gordh, Jr. and L. Lum, *Arc-smooth continua*, Trans. Amer. Math. Soc. **265** (1981), 545–561.

[43] H. M. Gehman, *Concerning the subsets of a plane continuous curve*, Ann. of Math. **27** (1925), 29–46.

[44] J. T. Goodykoontz, Jr., *Some functions on hyperspaces of hereditarily unicoherent continua*, Fund. Math. **95** (1977), 1–10.

[45] G. R. Gordh, Jr., *Aposyndesis in hereditarily unicoherent continua*, General Topology and Modern Analysis, L. F. McAuley and M. M. Rao (editors), Academic Press 1981, pp. 53–63.

[46] G. R. Gordh, Jr. and L. Lum, *Monotone retracts and some characterizations of dendrites*, Proc. Amer. Math. Soc. **59** (1976), 156–158.

[47] J. de Groot and R. J. Wille, *Rigid continua and topological group-pictures*, Arch. Math. (Basel) **9** (1958), 441–446.

[48] A. Illanes, *Characterizing dendrites by deformation retractions*, Topology Proc.**21** (1996), 129–141.

[49] A. Illanes, *A characterization of dendrites with the periodic-recurrent property*, preprint.

[50] H. Kato, *Generalized homogeneity of continua and a question of J. J. Charatonik*, Houston J. Math.**13** (1987), 51–63.

[51] H. Kato, *On problems of H. Cook*, Topology Appl. **26** (1987), 219–228.

[52] H. Kato, *A note on periodic points and recurrent points of maps of dendrites*, Bull. Austral. Math. Soc. **51** (1995), 459–461.

[53] B. Knaster , *Un continu dont tout sous-continuest indecomposable* , Fund. Math. , **3** (1922), 247 - 286 .

[54] P. Krupski, *Continua which are homogeneous with respect to continuity*, Houston J. Math. **5** (1979), 345–356.

[55] K. Kuratowski , Topology, Vol. II , Academic Press, New York, N.Y. , 1968 .

[56] A. Lelek, *On plane dendroids and their end points in the classical sense*, Fund. Math.**49** (1961), 301–319.

[57] L. Lum, *Weakly smooth dendroids*, Fund. Math. **83** (1974), 111–120.

[58] L. Lum *A characterization of local connectivity in dendroids*, Studies in topology (Proc. Conf. Univ. North Carolina, Charlotte, NC 1974, Academic Press, New York 1975, pp. 331–338.

[59] T. Maćkowiak, *Continuous mappings on continua*, Dissertationes Math. (Rozprawy Mat.)**158** (1979), 1–95.

[60] W. Makuchowski , *On open light mappings* , Comment. Math. Univ. Carolinae , **35,4** (1994), 779 - 788 .

[61] S. Mazurkiewicz, *Sur les continus homogènes*, Fund. Math. **5** (1924), 137–146.

[62] K. Menger, Kurventheorie, Teubner Verlag, Leipzig and Berlin 1932 (reprinted by Chelsea Publ. Co., Bronx, NY 1967).

[63] E. W. Miller, *Solution of the Zarankiewicz problem*, Bull. Amer. Math. Soc.**38** (1932), 831–834.

[64] S. B. Nadler, Jr., Hyperspaces of sets, M. Dekker 1978.

[65] S. B. Nadler, Jr., Continuum theory: an introduction, M. Dekker 1992.

[66] S. B. Nadler, Jr. and L. E. Ward, Jr., *Concerning continuous selections*, Proc. Amer. Math. Soc.**25** (1970), 369–374.

[67] J. Nikiel, *A characterization of dendroids with uncountably many end points in the classical sense*, Houston J. Math. **9** (1983), 421–432.

[68] J. Nikiel, *On Gehman dendroids*, Glasnik Mat. **20 (40)** (1985), 203–214.

[69] J. Nikiel, *Topologies on pseudo-trees and applications*, Mem. Amer. Math. Soc. **82** No. 416 (1989), 1–116.

[70] E. D. Nix, *Research problem 14*, Bull. Amer. Math. Soc. **76** (1970), 975–976.

[71] V. Neumann-Lara and I. Puga-Espinosa, *Shore points and dendrites*, Proc. Amer. Math. Soc.**118** (1993), 939–942.

[72] M. A. Owens, *Unicoherence at subcontinua*, Topology Appl. **22** (1986), 145–155.

[73] R. L. Plunkett, *A fixed point theorem for continuous multivalued transformations*, Proc. Amer. Math. Soc. **7** (1956), 160–163.

[74] L.A. Steen, J.A.Jr. Seebach , Counterexamples in topology , Springer-Verlag, New York , 1978 .

[75] L. E. Ward, Jr., *Characterization of the fixed point property for a class of set-valued mappings*, Fund. Math. **50** (1961), 159–164.

[76] T. Ważewski, *Sur les courbes de Jordan ne renfermant aucune courbe simple fermée de Jordan*, Annales de la Société Polonaise de Mathématique **2** (1923), 49–170.

[77] P. A. White, *r-regular convergence spaces*, Amer. J. Math. **66** (1944), 69–96.

[78] G. T. Whyburn, *Analytic topology*, Amer. Math. Soc. Colloq. Publ. 28, Providence 1942 (reprinted with corrections 1971).

[79] G.T. Whyburn , *Topological characterization of the Sierpinski curve* , Fund. Math. , **45** (1958), 320 - 324 .

[80] X. D. Ye, *The centre and the depth of the centre of a tree map*, Bull. Austral. Math. Soc. **48** (1993), 347–350.

[81] K. Zarankiewicz, *Problem 37*, Fund. Math. **7** (1925), 381.